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APPLICATION OF FUZZY SET THEORY FOR SELECTING AMT UNDER MULTIPLE CRITERIA APPROACH

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Abstract Advanced Manufacturing Technology (AMT) is the key factor for the future of any manufacturing units in today's highly competitive and changing environment. The inability of conventional justification methods to consider the non-quantifiable factors makes them inappropriate for most strategic decision-making models. In this paper, a distinct methodology is proposed for the selecting of AMT based on Fuzzy set theory under multiple criteria decision-making methodology to obtain fuzzy suitability indices. The selection of AMT is based on the ranking values of suitability indices. The problem considers subjective selection criteria as level of flexibility, quality, information and objective criteria as investment cost. Fuzzy linguistic variables are used as an approach for handling inexact data and yet to work in a mathematically strict and vigorous way. The proposed methodology considers the rating attitudes of decision-makers and trade-off among various selection criteria in the aggregation process to assure an accurate selection policy.

Keywords: Fuzzy set, AMT, Multi-criteria approach, Suitability indices.

INTRODUCTION

The profitability and productivity of any organization depends on the realization and timely selection of accurate AMT. New technology is the key to survival for present organizations in this competitive environment [Madu and Georgntzas, 1991]. Advanced technologies spread across the activities from factory operation to marketing and service. If these are properly selected and used, they can enhance productivity, thereby, boosting up to achieve better competitive posture. Advanced technologies are expensive and the selection should be carefully made considering all influencing subjective as well as objective parameters to survive in this competitive and turbulent environment. The selection of AMT depends on (1) technical factors that deal with the appropriateness of the technology; (2) structural factors that deal with the coordination, information and control and (3) strategic factors that deal with the management of resources and uncertainty [Noori, 1990]. Although research papers are available on the selection of new technology based on the cost accounting technique like net present value, internal rate of return etc. These approaches have considered only the quantitative factors. Many very important subjective factors such as competitive advantage and social issues. AMT selection problem deals with the appropriate advanced technology selection to allow greater productivity and certainty. Owing to the unstructured nature of the problem, many researchers have proposed various approaches, which have not been very successful to deal with the qualitative factors associated with the problem. Regardless of the type of data, there is an element of vagueness or fuzziness in it. Traditional selection methods are based on quantitative analysis of cost of investment and rate of return. Fuzzy set theory has been recently applied in various areas of production management [karwaski, 1986]. Most of the models and algorithms available in the literature are suitable for handling exact measure and crisp evaluation. In the real life, to evaluate AMT selection suitability, measures for the subjective criteria, e.g. level of flexibility, level of quality, level of information and skill of operators etc may not be preciously defined for the decision-makers [Deb et al, 2001]. Hence, the precision-based evaluation may not be practical. Besides, the evaluating data of the AMT selection suitability under different criteria as well as the weight of the criteria are often expressed in linguistic terms, e.g. very low, medium, high etc. Therefore, the present research work follows to integrate various linguistic assessments and weights to evaluate technology interrelation suitability and determine the best alternative selection order by using fuzzy multi-criteria decision-making methodology.

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FUZZY SET AND SYSTEM

The fuzzy set theory was introduced by Zadeh(1965) to deal with problems in which the absence of preciously defined criteria is involved. Formally, if $X=\{x\}$ is a set of objects, then the fuzzy set A on X is defined by its membership function $f_A(x)$ which assignees to each element $x \varepsilon X$ a real number in the interval { 0, 1 }which represents the grade of membership of x in A. Thus A can be written as: $A=\{(f_A(x)/x)|x \varepsilon X\};X\rightarrow[0,1]$. Trapezoidal fuzzy number can be denoted by $(\alpha,\beta,\gamma,\delta)$ as shown in figure 1. Its membership function $f_A(x):R\rightarrow[0,1]$ is written as:

$f_A(x)=(x-\alpha)/(\beta-\alpha)$ for $\alpha \leq x \leq \beta$

$=1$ for $\beta \leq x \leq \gamma$

$=(x-\delta)/(\gamma-\delta)$ for $\gamma \leq x \leq \delta$

$=0$ otherwise

With this notation and by the extension principle

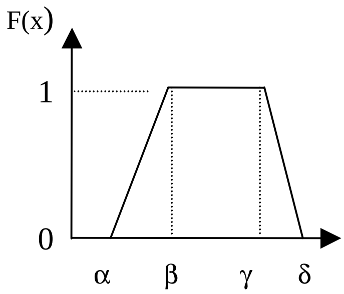

Fig. 1. Membership function of a trapezoidal fuzzy number.

proposed by Zadeh(1965) the extended algebraic operations on trapezoidal fuzzy numbers can be expressed as :

- Changing sign: $-(\alpha,\beta,\gamma,\delta)=(-\alpha,-\beta,-\gamma,-\delta)$
- Addition: $(\alpha_1,\beta_1,\gamma_1,\delta_1)\oplus(\alpha_2,\beta_2,\gamma_2,\delta_2)=(\alpha_1+\alpha_2, \beta_1+\beta_2, \gamma_1+\gamma_2, \delta_1+\delta_2)$
- Subtraction: $(\alpha_1,\beta_1,\gamma_1,\delta_1)\Theta(\alpha_2,\beta_2,\gamma_2,\delta_2)=(\alpha_1-\delta_2, \beta_1-\gamma_2, \gamma_1-\beta_2, \delta_1-\alpha_2)$
- Multiplication: $(\alpha_1,\beta_1,\gamma_1,\delta_1)\otimes(\alpha_2,\beta_2,\gamma_2,\delta_2)\cong(\alpha_1\alpha_2, \beta_1\beta_2, \gamma_1\gamma_2, \delta_1\delta_2)$
- Division: $(\alpha_1,\beta_1,\gamma_1,\delta_1)\varnothing(\alpha_2,\beta_2,\gamma_2,\delta_2)\cong(\alpha_1/\delta_2, \beta_1/\gamma_2, \gamma_1/\beta_2, \delta_1/\alpha_2)$
  for $\alpha_1\geq0$, $\alpha_2\geq0$.

Trapezoidal fuzzy numbers are easy to use and interpret. For example, 'approximately equal to 24' can be represented by (20, 24, 24, 30). The concept of linguistic variable is very useful in dealing with situations, which are too complex or too ill defined to be reasonably described in conventional quantitative expressions. A linguistic variable is a variable whose values are words in natural or artificial language. The approximate reasoning of fuzzy set theory can represent the Linguistic value [Zimmerman, 1987]. For example, the linguistic variable of weight may be considered as {VL, L, M, H, and VH}.

PROBLEM FORMULATIO AND PROCEDURE

In this paper, AMT selection under a fuzzy environment is proposed. The subjective (qualitative) as well as objective (quantitative) criteria associated with the problem are assigned by the decision makers within approximated information value available to determine the ranking of the alternatives

Model generation and selection criteria

The concept of hierarchical structure analysis with two distinct levels is used in this paper. The first level is to evaluate the fuzzy importance of the decision criteria (e.g. level of flexibility, safety, quality, supervision, working condition, investment cost etc). The hierarchical structure of the different factors for facility selection order is shown in figure 2. The second level is to assign to various alternatives under each decision criteria. A group of 'm' decision makers is assumed to employ rating sets to evaluate preference information. The decision makers assess the suitability of 'n' alternatives under each criterion. Let R_{ijk} be the rating assigned to alternative (i) by DM (j) for criteria (k). Similarly W_{kj} be the weight given to criteria (k) by DM (j). Thus the committee has to first aggregate the ratings R_{ijk} for each alternative versus each criterion to form the rating R_{ik}. Each aggregated R_{ik} for i=1,n; k=1,p; can further be weighted by a weight W_k according to the relative importance of the criteria. The fuzzy suitability index F_i of each alternative can be obtained by aggregating R_{ik} and W_k for all selection criteria to form a suitability vector. Finally, applying maximizing and minimizing fuzzy number the corresponding ranking values of the fuzzy suitability indices are obtained to decide about the best selection based on highest-ranking value.

Preference rating system

The preference rating adopted in the present problem are fuzzy members and linguistic variable values. The DM employs linguistic weighting set W={VL, L, M, H, VH} to evaluate the importance of the criteria through a designed rating scale as shown in figure 3 in the range of [0,1] whose membership functions of the linguistic values are shown as following trapezoidal fuzzy number and membership functions:

VL(0,0,0,0.3); L(0,0.3,0.3,0.5); M(0.2,0.5,0.5,0.8); H(0.5,0.7,0.7,1); VH(0.7,1,1,1).

The decision makers (DM) also employ a linguistic rating set R={VP, P, F, G, VG} to evaluate the suitability of

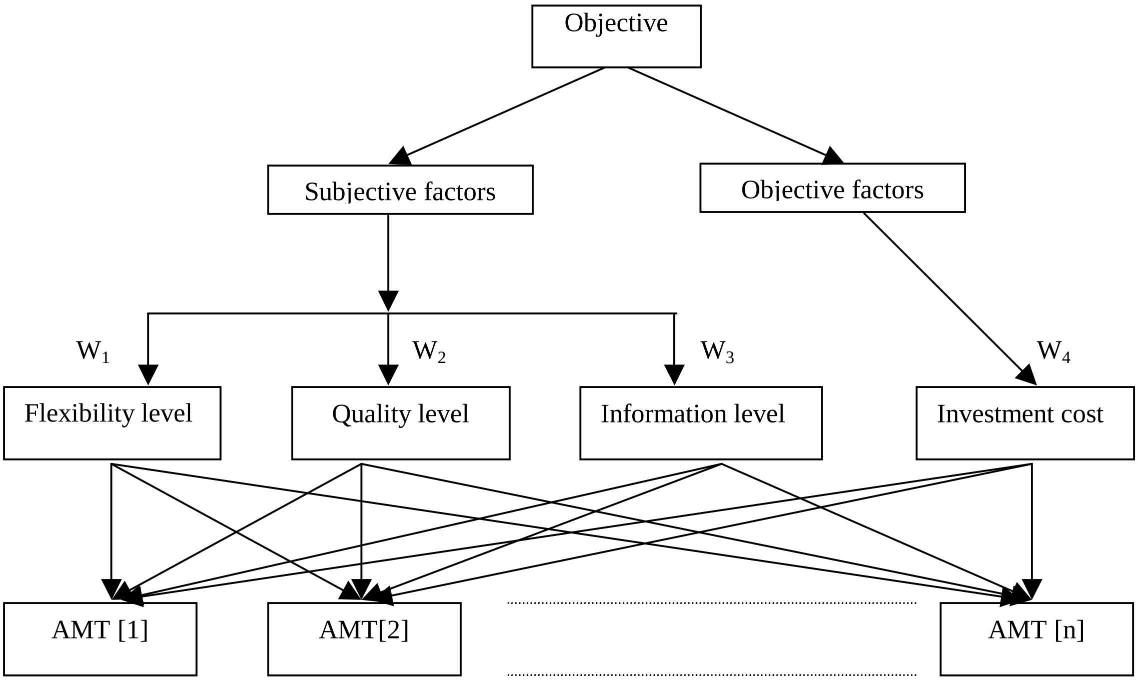

Fig. 2. Hierarchical structure with four criteria and different AMT.

VERYLOW(VL) : (0, 0, 0, 0.3)

$$f_W(x) = 1 - 10x/3 \qquad 0 \le x \le 0.3$$

LOW(L) :(0, 0.3, 0.3, 0.5)

$$f_W(x) = \begin{cases} 10x/3 & 0 \le x \le 0.3 \\ 5/2 - 5x & 0.3 \le x \le 0.5 \end{cases}$$

MEDIUM(M) :(0.2, 0.5, 0.5, 0.8)

$$f_W(x) = \begin{cases} 10x/3 - 2/3 & 0.2 \le x \le 0.5 \\ 8/3 - 10x/3 & 0.5 \le x \le 0.8 \end{cases}$$

HIGH(H) :(0.5, 0.7, 0.7, 1)

$$f_W(x) = \begin{cases} 5x - 5/2 & 0.5 \le x \le 0.7 \\ 10/3 - 10x/3 & 0.7 \le x \le 1 \end{cases}$$

VERYHIGH(VH) :(0.7, 1, 1, 1)

$$f_W(x) = 10x/3 - 7/3 \qquad 0.7 \le x \le 1$$

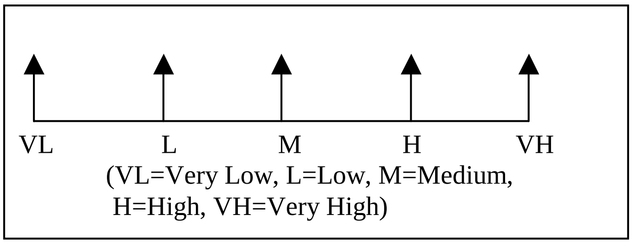

Fig. 3. Weighing scale used for the assessment of different criteria

various alternatives versus each subjective criterion. The membership function of linguistic ratings are represented in figure 4 and the trapezoidal fuzzy numbers are written as:

VP (0,0,0,0.2); P (0,0,0.2,0.4); F (0,0.2,0.4,0.6); G(0.2,0.4,0.6,0.8); VG (0.4,0.6,0.8,1)

In order to ensure compatibility between fuzzy cost values of the objective criterion and linguistic ratings of subjective criteria; fuzzy cost values must be converted to dimensionless indices. The alternative with the minimum cost value should have the maximum rating. The rating of alternative (i) for objective criterion can be written as:

$$RF_i = \{ F_i \otimes [F_1^{-1} \oplus F_2^{-1} \oplus F_3^{-1} \oplus \ldots\ldots\ldots \oplus F_n^{-1}]\}^{-1} \quad \ldots\ldots..(1)$$

Where F_i is the value of cost for alternative (i).

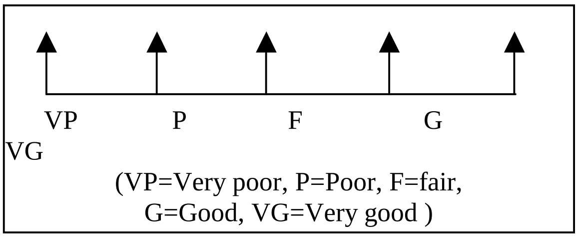

Fig. 4 Rating scale for linguistic variable set R.

Thus, RFi =$R_{i\,p}$.

VERYPOOR(VP):(0,0,0,0.2)

$f_R(x)=1-5x \qquad 0 \le x \le 0.2$

POOR(P):(0,0.2,0.2,0.4)

$$f_R(x)=\begin{cases} 5x & 0 \le x \le 0.2 \\ 2-5x & 0.2 \le x \le 0.4 \end{cases}$$

FAIR(F):(0.3,0.5,0.5,0.7)

$$f_R(x)=\begin{cases} 5x-3/2 & 0.3 \le x \le 0.5 \\ 7/2-5x & 0.5£x£0.7 \end{cases}$$

GOOD(G):(0.6,0.8,0.8,1)

$$f_R(x)=\begin{cases} 5x-3 & 0.6 \le x \le 0.8 \\ 5-5x & 0.8 \le x \le 1 \end{cases}$$

VERYGOOD(VG):(0.8,1,1,1)

$f_R(x)=5x-4 \qquad 0.8 \le x \le 1$

Aggregation of fuzzy assessment

The mean operator is most commonly used to aggregate the decision makers' fuzzy assessments.

Let $R_{ijk}=(r_{ijk}^1, r_{ijk}^2, r_{ijk}^3, r_{ijk}^4)$, for i=1,n; j=1,m; k=1,p. be the linguistic rating assigned to alternative (i) by DM (j) for criterion (k) and let $W_{kj}=(w_{kj}^1, w_{kj}^2, w_{kj}^3, w_{kj}^4)$, for k=1,p; j=1,m; be the linguistic weight given to subjective criteria (1,2,…,p-1) and the objective criterion (p) by DM (j). The average linguistic rating and weight are written as:

$R_{ik}=(1/n)\otimes[R_{ik1} \oplus R_{i\,k2} \oplus \ldots \oplus R_{ikm}]$ for i=1,n; for k=1,p-1.

$=(r_{ip}^1, r_{ip}^2, r_{ip}^3, r_{ip}^4)$ for i=1.n; k=p.

and $W_k=(1/m)\otimes[w_{k1}\oplus w_{k2}\oplus \ldots \oplus w_{km}]$ for k=1,p.

then $R_{ik}=(r_{ik}^1, r_{ik}^2, r_{ik}^3, r_{ik}^4)$ …………………..(2)

and $W_k=(w_k^1, w_k^2, w_k^3, w_k^4)$ …………………. (3)

R_{ik} and W_k are further aggregated by averaging the corresponding products over all the criteria. The fuzzy suitability index of the i th alternative can be obtained by standard arithmetic method written as:

$F_i=(i/p)\otimes[(R_{i1}\otimes W_1)\oplus(R_{i2}\otimes W_2)\oplus\ldots\oplus(R_{ip}\otimes W_p)]$…....(4)

which provides a trapezoidal fuzzy number as given by

$F_i=(\alpha_i, \beta_i, \gamma_i, \delta_i)$.

Ranking values of alternatives:

The ranking values of the alternatives are determined by using Madu and Georgntzas maximizing set (M) and minimizing set (N) [Chen, 1985] as given below:

$M=\{(x, f_M(x)) \mid x \varepsilon R\}$ with membership function values as

$f_M(x)=[(x-x_1)/(x_2-x_1)]^k$ for $x_2 \ge x \ge x_1$

=0 otherwise.

The membership function for minimizing set is given as:

$F_N(x)=[(x-x_2)/(x_1-x_2)]^k$ for $x_2 \ge x \ge x_1$

=0 otherwise.

Where k>0, x_1=inf D, x_2=sup D, $D=U_{i=1,n}D_i$, $D_i=\{x|f_{Fi}(x)>0\}$

The value of k depends on decision makers preference. The ranking value of fuzzy suitability index can be obtained by the ranking value of trapezoidal fuzzy number $F_i=(\alpha,\beta,\gamma,\delta)$ with the help of equation,

$V(F_i)=[(\delta_i-x_1)/((x_2-x_1)-(\gamma_i-\delta_i))+1-(x_2-\alpha_i)/((x_2-x_1)+(\beta_i-\alpha_i))]/2$ …………………………………………...(5)

The ranking values of fuzzy suitability indices of n alternatives are determined. Based on the ranking values, the decision maker can easily make the best MHE selection for the alternative having highest-ranking value.

STEPS OF ALGORITHM

Step1 Form a decision making group and decide the selection criteria for the different alternatives of AMT.

Step2 Define the appropriate preference ratings for the importance of the alternative AMT selection criteria.

Step3 Find the suitable preference ratings for the suitability of alternatives versus differ4ent criteria.

Step4 Find the aggregated weightings W_k and aggregated fuzz ratings (R_{ik}) of each alternative under all the criteria.

Step5 Assign ratings (RF_i) to the objective criteria.

Step6 Aggregate R_{ik} and W_k with respect to each criterion to obtain fuzzy suitability indices of all alternatives.

Step7 Determine the ranking value (V (F_i)) associated with each alternative's fuzzy suitability index F_i.

Step8 Select the alternative ATM with the maximum ranking value.

EXAMPLES

In this section, a hypothetical AMT selection problem is designed to demonstrate the application of the procedure. The problem statement is given below:

Number of alternatives (AMT)=4, number of decision makers (DMs)=2 and number of subjective criteria =3 (flexibility, quality and information) and one objective criterion as investment cost. The decision makers' assessment table for alternatives versus criteria and weight assigned to four criteria are shown in table 1. The aggregated weighing (W_k) of the decision maker is obtained by using equation (3) as given below:

W_1=(0.385, 0.675, 0.675, 0.865);
W2=(0.515, 0.750, 0.750, 0.950);
W_3=(0.624, 0.850, 0.850, 0.956);
W_4=(0.655, 0.925, 0.925, 1.000).

Table 1. DMs assessment for alternative Vs criteria and weight Vs criteria

Criteria(K_i) Decision Maker (D)	Flexibility D1 D2	Quality D1 D2
Weightage	M VH	H VH
Alternative1	F G	P F
Alternative2	G F	F G
Alternative3	P G	G VG
Alternative4	VG G	F F
Criteria(K_i) Decision Maker (D)	Information D1 D2	Investment Cost ($\$*10^5$) D1 D2
Weightage	VH H	VH H
Alternative1	G VG	(18,22,28,32)
Alternative2	F P	(22,26,32,38)
Alternative3	P F	(22,25,28,33)
Alternative4	P G	(16,20,28,33)

(VL=Very low, L=Low, M=Medium, H=High, VP=Very poor, P=Poor, =Fair, G=Good)

Table 2 Average fuzzy subjective and objective rating

K_i	Flexibility	Quality
A1	(.23,.38,.58,.74)	(.17,.36,.44,.65)
A2	(.52,.71,.76,.93)	(.22,.38,.45,.65)
A3	(.22,.43,.62,.82)	(.28,.48,.68,.89)
A4	(.26,.38,.47,.64)	(.24,.36,.47,.69)
K_i	Information	Investment Cost
A1	(.51,.69,.78,.92)	(.22.25,.32,.36)
A2	(.34,.54,.72,.92)	(.24,.29,.41,.54)
A3	(.32,.48,.62,.84)	(.26,.32,.43,.56)
A4	(.20,.32,.46,.62)	(.23,.27,.36,.48)

Table 3. Fuzzy suitability indices and ranking values

Altern atives	Fuzzy suitability indices of the alternatives	Ranking values
AMT1	(0.170, 0.322,0.463,0.620)	0.386
AMT2	(0.182,0.344,0.480,0.673)	0.446
AMT3	(0.164,0.375,0.462,0.648)	0.431
AMT4	(0.156,0.362,0.485,0.612)	0.427

By using equation (2) with reference to the DMs assessment of alternatives versus each criteria as shown in table 1, the average evaluation of alternatives for subjective criteria (R_{ik} for k=1,p-1) are computed. Similarly by using equation (1) the fuzzy objective rating (R_{ik} for k=p) is obtained and they are presented in table 2. Fuzzy suitability indices are obtained after aggregating R_{ik} and W_k by averaging the corresponding products over all the criteria by using equation (4). The ranking values of the fuzzy suitability indices are obtained by using equation (5) and presented in table 3. The ranking order of fuzzy suitability for the four alternatives is AMT1, AMT2, AMT3 and AMT4. Therefore, the best selection is AMT2 alternative under present multiple criteria approach.

DISCUSSON AND CONCLUSION

In this paper, a decision procedure is proposed to solve the MHE selection under fuzzy environment. The conventional approaches are less sensitive in making effective decision when the assessments of alternatives versus criteria and the importance weights are given in linguistic terms. The proposed methodology considers both objective and subjective factors in such a manner that the viewpoints of total decision-making body can be expressed without any constraints. Thus by conducting fuzzy linguistic assessments and fuzzy objective assessments, the DMs can have the final ranking of the alternatives automatically.

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